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MODELING OF VERTICAL TURBULENT EXCHANGE IN A STRATIFIED NEAR-WALL FLOW

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A modified model of turbulence is proposed to describe the processes of vertical transport in inhomogeneous turbulent flows. This model includes algebraic relations for the Reynolds stresses and turbulent-exchange coefficients. Using this model, the growth of the depth of a mixed layer under the action of the wind load in neutral and stable stratified near-wall flows has been predicted. The calculation results for a stable stratified flow that were obtained using the modified and standard two-parametric models of turbulence are compared with experimental data.

Mathematical modeling of the temperature regime in stratified lakes and water storages requires the use of adequate approximations of vertical turbulent exchange. The $(E–\varepsilon)$ model of turbulence is frequently used now to define the effective transport coefficients in transport equations for velocity and temperature (concentration). Most of its variants, however, do not take into account the damping of the vertical fluctuations of velocity near the bottom and the free surface, which makes the obtaining of reliable characteristics of turbulence in the corresponding regions rather problematic. A correct modeling of the parameters of turbulent exchange in the region of a thermal wedge remains an important problem. A solution of this problem will make it possible to describe adequately the transport of heat and matter between the surface and bottom regions in stratified reservoirs. An extended variant of the $(E–\varepsilon)$ model of turbulence that involves algebraic relations for determination of the Reynolds stresses is considered in the present paper.

The improved model of turbulence was tested by solving the problem of deepening the mixed layer in constant-density and stratified fluids under the action of wind stresses.

Mathematical Model. In modeling hydrophysical processes in a stable stratified flow we used a hydrostatic approximation and an assumption of horizontal homogeneity (averaging of hydrodynamic quantities in the horizontal plane). If the vertical component W of the mean-velocity vector is absent, the processes of turbulent transport of heat (salinity) and momentum are described by the equations

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\Big[\lambda \frac{\partial T}{\partial z} - \langle w\theta\rangle\Big]; \tag{1}$$

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial z}\Big[\nu \frac{\partial U}{\partial z} - \langle uw\rangle\Big], \qquad \frac{\partial V}{\partial t} = \frac{\partial}{\partial z}\Big[\nu \frac{\partial V}{\partial z} - \langle vw\rangle\Big]. \tag{2}$$

Here T and θ are the mean and fluctuating components of the temperature (salinity) of water, $\mathbf{U} = (U, V)$ and $\mathbf{u} = (u, v)$ are the vectors of the mean and fluctuating components of the flow velocity in the horizontal plane, w is the vertical component of the velocity fluctuation, $\langle\cdots\rangle$ denotes the averaging over an ensemble of realizations, λ and ν are the coefficients of molecular transport, t is the time, and the vertical coordinate z is directed upwards and counted from the bottom.

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The gradient relations $\langle w\theta\rangle = -\lambda_t(\partial T/\partial z)$, $\langle uw\rangle = -\nu_t(\partial U/\partial z)$, and $\langle vw\rangle = -\nu_t(\partial V/\partial z)$ in combination with the standard (E–ε) model of turbulence lead to the closure of Eqs. (1) and (2) [1-3]. Here λ_t and ν_t are the turbulent temperature diffusivity (diffusion) and viscosity. The model of turbulent transport can be refined by involving differential transport equations for turbulent fluxes of momentum and heat (second-order correlations of thermohydrodynamic fields) [4-6]:

$$L(\langle u_i u_j\rangle) = -\langle u_i u_k\rangle \frac{\partial U_j}{\partial x_k} - \langle u_j u_k\rangle \frac{\partial U_i}{\partial x_k} + \frac{g_i}{\rho^*}\langle \rho' u_j\rangle + \frac{g_j}{\rho^*}\langle \rho' u_i\rangle + \pi_{ij} - \varepsilon_{ij}; \tag{3}$$

$$L(\langle u_i\theta\rangle) = -\langle u_k\theta\rangle \frac{\partial U_i}{\partial x_k} - \langle u_i u_k\rangle \frac{\partial T}{\partial x_k} + \frac{g_i}{\rho^*}\langle \rho'\theta\rangle + \pi_{i\theta} - \varepsilon_{i\theta}, \tag{4}$$

where $g_i = (0, 0, -g)$ is the vector of gravity acceleration, ρ^* is the water density averaged over the entire flow, ρ and ρ' are the mean (over the ensemble of realizations) and fluctuating components of the density, and the density is found from the equation of state $\rho = \rho(T)$. The sources of buoyancy in the Boussinesq approximation [$|\rho(x_i,t) - \rho^*| \ll \rho^*$ and $\partial p_0/\partial x_i = \rho^* g_i$, where p_0 is the hydrostatic pressure] contain the correlations $\langle\rho' u_i\rangle$ and $\langle\rho'\theta\rangle$ determined from the transport equations

$$\begin{gathered} L(\langle \rho' u_i\rangle) = -\langle \rho' u_k\rangle \frac{\partial U_i}{\partial x_k} - \langle \rho' u_i\rangle \frac{\partial \rho}{\partial x_k} + \frac{g_i}{\rho^*}\langle \rho'^2\rangle + \pi_{i\rho} - \varepsilon_{i\rho}, \\ L(\langle \rho'^2\rangle) = -2\langle \rho' u_k\rangle \frac{\partial \rho}{\partial x_k} - \varepsilon_\rho, \quad L(\langle \rho'\theta\rangle) = -\langle \rho' u_k\rangle \frac{\partial T}{\partial x_k} - \langle u_k\theta\rangle \frac{\partial \rho}{\partial x_k} - \varepsilon_{\rho\theta}. \end{gathered} \tag{5}$$

The convective-diffusion operator in Eqs. (3)–(5) has the form

$$L(\Phi) = \frac{\partial \Phi}{\partial t} + U_k \frac{\partial \Phi}{\partial x_k} - D(\Phi),$$

where the terms $D(\Phi)$ describe the processes of molecular and turbulent diffusion (contain triple correlations). The dissipative terms are modeled as [4-6]

$$\varepsilon_{ij} = \frac{2}{3}\varepsilon\delta_{ij}, \quad \varepsilon_{i\theta} = \varepsilon_{i\rho} = 0, \quad \varepsilon_\rho = \frac{1}{R}\frac{\varepsilon}{E}\langle\rho'^2\rangle, \quad \varepsilon_{\rho\theta} = \frac{1}{R}\frac{\varepsilon}{E}\langle\rho'\theta\rangle.$$

The expressions for the correlations with pressure fluctuations π_{ij}, $\pi_{i\theta}$, and $\pi_{i\rho}$, which take into account the effects of the mean velocity shear, Archimedes buoyancy forces, and the free surface, were borrowed from [5-7].

The differential model (3)–(5) is simplified and becomes an algebraic model if we ignore the terms $L(\Phi)$ in the equations for the tangential Reynolds stresses $\langle u_i u_j\rangle$ ($i \neq j$) and correlations $\langle u_i\theta\rangle$, $\langle\rho' u_i\rangle$, $\langle\rho'\theta\rangle$, and $\langle\rho'^2\rangle$, and assume the advection and diffusion terms in the equations for the normal Reynolds stresses (in particular, for $\langle w^2\rangle$) to be proportional to the corresponding terms of the equation for the turbulence energy $E = \langle u_i u_i\rangle/2$, as in [6, 7]. Under the assumption of horizontal homogeneity and for $W = 0$ the resultant relations for the second-order correlations are

$$\langle w\theta\rangle = E\,\frac{-\langle w^2\rangle(\partial T/\partial z) + [1 - C_{2\theta}(1 - C'_{2\theta}f)](-(g/\rho^*)\langle\rho'\theta\rangle)}{(C_{1\theta} + C'_{1\theta}f)\varepsilon}; \tag{6}$$

$$\langle uw\rangle = E\,\frac{[1 - C_2(1 - (3/2)C'_2 f)](-\langle w^2\rangle(\partial U/\partial z) - (g/\rho^*)\langle\rho' u\rangle)}{(C_1 + (3/2)C'_1 f)\varepsilon}; \tag{7}$$

$$\langle vw\rangle = E\,\frac{[1 - C_2(1 - (3/2)C'_2 f)](-\langle w^2\rangle(\partial V/\partial z) - (g/\rho^*)\langle\rho' v\rangle)}{(C_1 + (3/2)C'_1 f)\varepsilon}; \tag{8}$$

$$\langle w^2\rangle = \frac{2}{3}E\Big(1 - \frac{2C'_1 f\varepsilon + [1 - C_2(1 - 2C'_2 f)](P - 3G)}{(C_1 + 2C'_1 f)\varepsilon + P - \varepsilon}\Big); \tag{9}$$

$$\langle\rho'\theta\rangle = R(E/\varepsilon)\Big[-\langle w\theta\rangle\frac{\partial\rho}{\partial z} - \langle\rho' w\rangle\frac{\partial T}{\partial z}\Big]; \tag{10}$$

$$\langle \rho' u \rangle = C_{1\theta}^{-1}(E/\varepsilon)\Big[-\langle uw\rangle \frac{\partial \rho}{\partial z} - (1 - C_{2\theta})\langle \rho' w\rangle \frac{\partial U}{\partial z}\Big]; \tag{11}$$

$$\langle \rho' v \rangle = C_{1\theta}^{-1}(E/\varepsilon)\Big[-\langle vw\rangle \frac{\partial \rho}{\partial z} - (1 - C_{2\theta})\langle \rho' w\rangle \frac{\partial V}{\partial z}\Big]; \tag{12}$$

$$\langle \rho' w \rangle = E\,\frac{-\langle w^2\rangle(\partial\rho/\partial z) + [1 - C_{2\theta}(1 - C'_{2\theta} f)](-(g/\rho^*)\langle \rho'^2\rangle)}{(C_{1\theta} + C'_{1\theta} f)\varepsilon}; \tag{13}$$

$$\langle \rho'^2 \rangle = -2R(E/\varepsilon)\langle \rho' w\rangle \frac{\partial \rho}{\partial z} \tag{14}$$

$$\Big(P = -\langle uw\rangle \frac{\partial U}{\partial z} - \langle vw\rangle \frac{\partial V}{\partial z} + G, \qquad G = -\frac{g}{\rho^*}\langle \rho' w\rangle\Big).$$

Here P and ε are the rates of production and dissipation of the turbulence energy, G is a source term caused by the action of Archimedes forces, and $C_1 = 1.8$, $C_2 = 0.6$, $C'_1 = 0.5$, $C'_2 = 0.3$, $C_{1\theta} = 3.0$, $C_{2\theta} = 0.5$, $C'_{1\theta} = 0.5$, $C'_{2\theta} = 0.3$, and $R = 0.8$ are the constants. The damping of the vertical fluctuations of velocity near the free surface ($z = H$) is taken into account by introducing into the algebraic relations (6)–(9) and (13) an empirical function $f(z)$ of the form [7]

$$f = C_f\,\frac{E^{3/2}}{\varepsilon}\Big[H - z + 0.04\,\frac{E_s^{3/2}}{\varepsilon_s}\Big]^{-1},$$

where $E_s = E|_{z=H}$, $\varepsilon_s = \varepsilon|_{z=H}$, and $C_f = 1/15$ is an empirical constant.

A successive substitution of (14) into (13), (13) into (11) and (12), (11) into (7), (12) into (8), and also (13) into (10) and (10) into (6) yields the gradient relations

$$\langle \rho' w\rangle = -d_t\Big(\frac{\partial \rho}{\partial z}\Big), \quad \langle w\theta\rangle = -\lambda_t\Big(\frac{\partial T}{\partial z}\Big), \quad \langle uw\rangle = -\nu_t\Big(\frac{\partial U}{\partial z}\Big), \quad \langle vw\rangle = -\nu_t\Big(\frac{\partial V}{\partial z}\Big),$$

$$\begin{aligned} d_t &= \frac{\langle w^2\rangle E}{(C_{1\theta} + C'_{1\theta} f)\varepsilon + [1 - C_{2\theta}(1 - C'_{2\theta} f)][2(g/\rho^*)R(E^2/\varepsilon)(-\partial\rho/\partial z)]},\\ \lambda_t &= \frac{\langle w^2\rangle E + [1 - C_{2\theta}(1 - C'_{2\theta} f)]R(E^2/\varepsilon)G}{(C_{1\theta} + C'_{1\theta} f)\varepsilon + [1 - C_{2\theta}(1 - C'_{2\theta} f)][(g/\rho^*)R(E^2/\varepsilon)(-\partial\rho/\partial z)]},\\ \nu_t &= \frac{[1 - C_2(1 - (3/2)C'_2 f)][\langle w^2\rangle E + (1 - C_{2\theta})C_{1\theta}^{-1}(E^2/\varepsilon)G]}{(C_1 + (3/2)C'_1 f)\varepsilon + [1 - C_2(1 - (3/2)C'_2 f)][(g/\rho^*)C_{1\theta}^{-1}(E^2/\varepsilon)(-\partial\rho/\partial z)]}, \end{aligned} \tag{15}$$

where d_t is the coefficient of turbulent mass transfer. The algebraic model (15) differs from that used in [4–6] by the presence of two relations for d_t and λ_t instead of one relation for the coefficient of turbulent heat transfer (mass transfer). This is due to the fact that the functions $\rho(T)$ and $\rho'(\theta)$ can be nonlinear (for example, if T is the temperature near the point of 0°C).

With (9) and (15) taking into account, the system of equations of turbulent transport acquires the same form as in [1–3]:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\Big[K_T\,\frac{\partial T}{\partial z}\Big]; \tag{16}$$

$$\frac{\partial \mathbf{U}}{\partial t} = \frac{\partial}{\partial z}\Big[K_U\,\frac{\partial \mathbf{U}}{\partial z}\Big]; \tag{17}$$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial z}\Big[K_E\,\frac{\partial E}{\partial z}\Big] + P - \varepsilon; \tag{18}$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial z}\Big[K_\varepsilon\,\frac{\partial \varepsilon}{\partial z}\Big] + \frac{\varepsilon}{E}(\hat{C}_1 P - \hat{C}_2\varepsilon), \tag{19}$$

where

$$P = \nu_t\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] + G, \qquad G = \frac{g}{\rho^*}\, d_t\, \frac{\partial \rho}{\partial z}.$$

The coefficients of the effective vertical transport of heat, momentum, and turbulence parameters are determined from the relations

$$K_T = \lambda + \lambda_t, \quad K_U = \nu + \nu_t, \quad K_E = \nu + C_s\,\frac{E}{\varepsilon}\,\langle w^2\rangle, \quad K_\varepsilon = \nu + C_\varepsilon\,\frac{E}{\varepsilon}\,\langle w^2\rangle.$$

The quantities λ_t, ν_t, and d_t, assigned in accordance with (15), are complex functions of U, T, E, and ε, in contrast to those used in [1–3]. The empirical coefficients and functions are $\hat{C}_1 = 1.55$, $\hat{C}_2 = 2(1 - 0.3\exp(-\mathrm{Re}_t^2))$ [$\mathrm{Re}_t = E^2/(\nu\varepsilon)$ is the turbulent Reynolds number] [1–3], $C_s = 0.22$, and $C_\varepsilon = 0.15$ [4].

The system (16)–(19) is supplemented by the boundary conditions

$$T = T_b, \qquad K_U\,\frac{\partial \mathbf{U}}{\partial z} = k_b|\mathbf{U}|\mathbf{U}, \qquad \frac{\partial E}{\partial z} = 0, \qquad \varepsilon = C_b\,\frac{E^{3/2}}{l_b} \tag{20}$$

at the bottom ($z = 0$) and

$$K_T\,\frac{\partial T}{\partial z} = 0, \qquad K_U\,\frac{\partial \mathbf{U}}{\partial z} = \frac{\boldsymbol{\tau}}{\rho}, \qquad \frac{\partial E}{\partial z} = k_\tau\left[\frac{|\boldsymbol{\tau}|}{\rho}\right]^{3/2}, \qquad \frac{\partial \varepsilon}{\partial z} = 0 \tag{21}$$

on the free surface ($z = H$). Here $C_b = 0.314$, $k_b = 0.014$, $k_\tau = 2.5$, l_b is the scale of roughness for $z = 0$ [1, 2], T_b is the temperature (salinity) of water at the bottom, and $\boldsymbol{\tau}$ is the wave load (shear stress) on the free surface of the flow. The parameters T_b and $\boldsymbol{\tau}$ can be different depending on the specific formulation of the problem.

The following profiles are considered as known for $t = 0$:

$$T(0,z) = T_0(z), \quad \mathbf{U}(0,z) = \mathbf{U}_0(z), \quad E(0,z) = E_0(z), \quad \varepsilon(0,z) = \varepsilon_0(z).$$

Numerical Implementation and Calculation Results. The proposed model of turbulence was used to calculate the evolution of turbulent motion in an open channel which was initiated by a wind load on the free surface. The nonlinear boundary-value problem (9) and (15)–(21) was solved numerically by the finite-difference method. The balance method [8, 9] was used to obtain implicit finite-difference schemes that approximate the differential problem. The system of algebraic finite-difference equations was solved by a scalar sweep method with iterations introduced because of nonlinearity. Calculations on successive uniform grids (each of them differs from the previous one by a twofold increase in the number of grid nodes N in the z direction) showed that for $N \sim 100$ the solution is almost independent of a further increase in the number of nodes; hence, we used $N = 100$. The time step was chosen such that no noticeable changes in the sought functions were observed with decreasing time step.

We consider the evolution of a shear flow in a circular channel under the action of a constant shear stress applied to the free surface of an initially quiescent fluid. The channel height is $H = 30$ cm. The wind load $\tau = |\boldsymbol{\tau}|$ in all the calculations was taken equal to 0.995 g/(cm·sec^2), which corresponded to experimental conditions [10]. The growth of the depth of the mixed layer in constant-density ($\partial\rho/\partial z = 0$) and initially stable stratified ($\partial\rho/\partial z = (\partial\rho/\partial z)_0 = 1.92\cdot 10^{-3}$ g/cm^4 for $t = 0$ [10]) fluids was calculated. The function T in (16) was the salinity (concentration of salt in water) linearly related to the density as $\rho(T) = \rho_0 + \alpha T$ (ρ_0 is the density of fresh water at a temperature of 13°C [10] and α is a constant coefficient). The molecular viscosity and diffusion (of salt in water) are $\nu = 0.01$ cm^2/sec and $\lambda = 1.2\cdot 10^{-5}$ cm^2/sec [10].

Figures 1–6 show calculation results obtained for the standard (E–ε) model [1–3] with turbulent-exchange coefficients $\nu_t = 0.09E^2/\varepsilon$ and $\lambda_t = d_t = 0.8\nu_t$ (dashed curves) and for the modified algebraic model (9) and (15) (solid curves).

Figure 1 shows the turbulent viscosity $\nu_t(z)$ as a function of the distance from the channel bottom. The lower values of $\nu_t(z)$ in the surface layer which were obtained using the modified model are caused by the damping effect of the free surface on velocity fluctuations. Mean-velocity profiles for a uniform fluid are shown in Fig. 2. The analysis of the numerical solutions shows that the mean-velocity distributions plotted

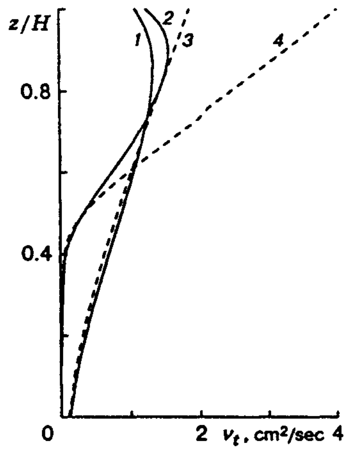

Fig. 1

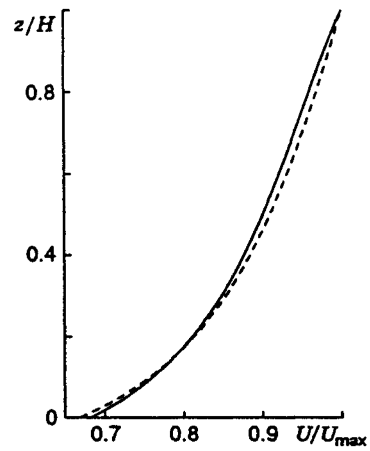

Fig. 2

Fig. 1. Turbulent viscosity distribution for t = 180 sec: constant-density flow (curves 1 and 3) and stratified flow (curves 2 and 4).

Fig. 2. Mean-velocity profiles in a homogeneous fluid obtained using the standard E–ε model (dashed curve) and the modified model (solid curve).

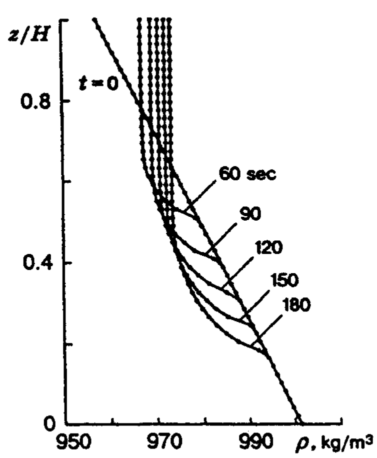

Fig. 3

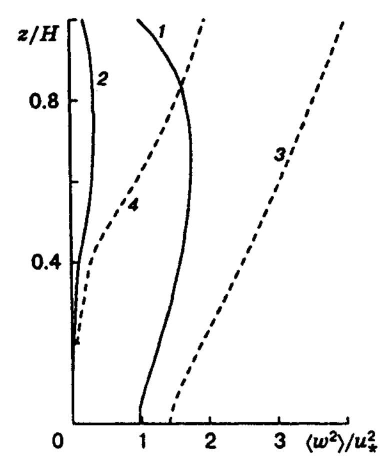

Fig. 4

Fig. 3. Dynamics of fluid-density behavior calculated using the modified model (the points indicate every other node of the grid).

Fig. 4. Distributions of vertical root-mean-square fluctuations of velocity for t = 180 sec: constant-density flow (curves 1 and 3) and stratified flow (curves 2 and 4).

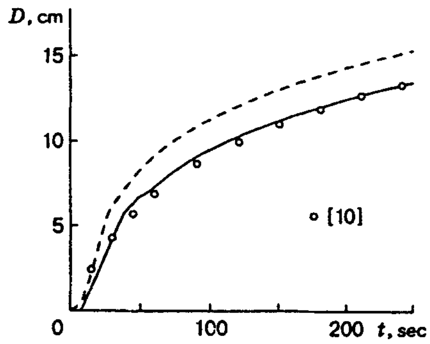

Fig. 5

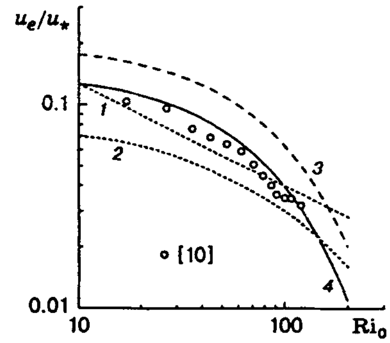

Fig. 6

Fig. 5. Dynamics of the mixed-layer thickness obtained using the standard (E–ε) model (dashed curve) and the modified model (solid curve).

Fig. 6. Entrainment velocity u_e/u_* versus the Richardson number Ri_0: curve 1 refers to the model with an algebraic relation for the length scale $L \sim E^{3/2}/\varepsilon \sim D$ [11], curve 2 refers to the model with a transport equation for L [11], curve 3 refers to the standard (E–ε)-model, and curve 4 refers to the modified model.

in Fig. 2 correspond actually to a steady-state solution of problem (9) and (15)–(21). The substitution of the standard model for the modified model leads to an insignificant difference in the calculated curves $U(z)$.

For the case of a linearly stratified fluid at the initial moment, the solution of problem (9) and (15)–(21) is unsteady within the entire interval $0 \leqslant t \leqslant 250$ sec. A turbulent shear flow that arises initially in the near-surface layer penetrates into the lower layers of the quiescent fluid and involves them in the turbulent motion. The process of deepening of the mixed layer is illustrated by the pattern of density variation in Fig. 3. This calculation was performed using the modified model of turbulence, and the calculations using the standard model predict similar results.

Distributions of the vertical root-mean-square fluctuations of velocity $\langle w^2 \rangle$ obtained using the standard and modified E–ε models for $t = 180$ sec are shown in Fig. 4. The curves $\langle w^2 \rangle$ for a turbulent constant-density flow are also shown here. Both models describe the suppression of turbulent fluctuations of velocity by stable stratification, while only the modified model (curves 1 and 2 in Fig. 4) describes the damping by the free surface of the fluid.

The dynamics of the mixed-layer thickness $D = D(t)$ in a stratified fluid that was calculated using two models of turbulence is shown in Fig. 5. The same figure shows an experimental dependence $D = D(t)$ from [10]. The lower boundary of the mixed layer in the calculations was assumed to be the value of z for which $\nu_t = 1$ cm^2/sec. The dependence $D = D(t)$ predicted by the modified model is in better agreement with the experiment than the dependence obtained using the standard model of turbulence, though a qualitative agreement is observed in both calculations.

Figure 6 shows the velocity of entrainment $u_e = dD/dt$ of the nonturbulent fluid into a turbulent region (of size D) as a function of the flow Richardson number $Ri_0 = g(\partial\rho/\partial z)_0 D^2/(2\rho^* u_*^2)$, where $u_* = \sqrt{\tau/\rho^*}$ is the friction velocity. The calculation results obtained using the standard and modified E–ε models, the models of turbulence from [11] (dashed curves), and also the measurement data of [10] are shown in the figure. The model from [11] includes transport equations for the second-order correlations without regard for the effect of the mean shear, buoyancy forces, and damping by the wall in the correlations with pressure fluctuations that enter in these equations, and a transport equation (or an algebraic relation) for the linear scale of turbulence $L \sim E^{3/2}/\varepsilon$ instead of the transport equation for dissipation ε. It is seen from Fig. 6 that the results obtained using the proposed model of turbulence are in better qualitative and quantitative agreement with experimental data than the results obtained using the standard E–ε model from [1–3] and the models from [11].

Results of the calculations performed give grounds for using the proposed model of the vertical turbulent exchange for numerical modeling of unsteady hydrophysical processes in deep stable stratified lakes and water storages, particularly when a more detailed description of the turbulent characteristics of the mixed surface layer is needed.

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